Let's Do Algebra Tiles

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Algebra Tiles

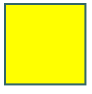
- Manipulatives used to enhance student understanding of subject traditionally taught at symbolic level.
- Provide access to symbol manipulation for students with weak number sense.
- Provide geometric interpretation of symbol manipulation.

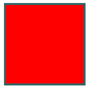
Algebra Tiles

- Support cooperative learning, improve discourse in classroom by giving students objects to think with and talk about.
- **When I listen, I hear.**
- **When I see, I remember.**
- **But when I do, I understand.**

Algebra Tiles

- Algebra tiles can be used to model operations involving integers.
- Let the small yellow square represent +1 and the small red square (the flip-side) represent -1.

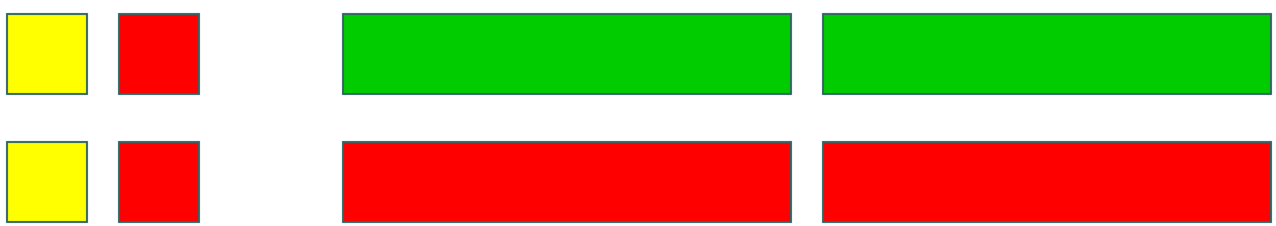




- The yellow and red squares are additive inverses of each other.

Zero Pairs

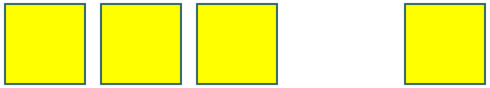
- Called zero pairs because they are additive inverses of each other.
- When put together, they cancel each other out to model zero.

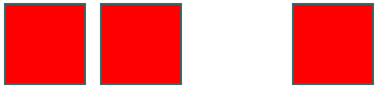


Addition of Integers

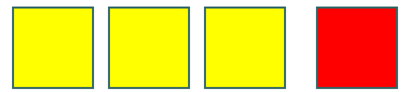
- Addition can be viewed as "combining".
- Combining involves the forming and removing of all **zero pairs**.
- For each of the given examples, use algebra tiles to model the addition.
- Draw pictorial diagrams which show the modeling.

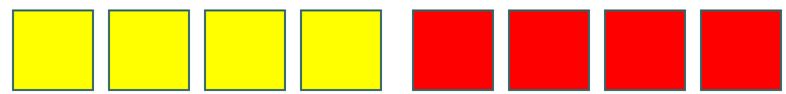
Addition of Integers

(+3) + (+1) = 

(-2) + (-1) = 

Addition of Integers

(+3) + (-1) = 

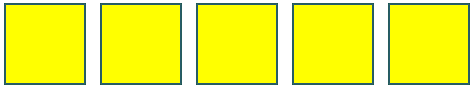
(+4) + (-4) = 

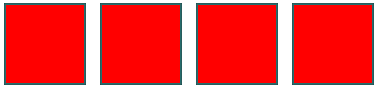
- After students have seen many examples of addition, have them formulate rules.

Subtraction of Integers

- Subtraction can be interpreted as "take-away."
- Subtraction can also be thought of as "adding the opposite."
- For each of the given examples, use algebra tiles to model the subtraction.
- Draw pictorial diagrams which show the modeling process.

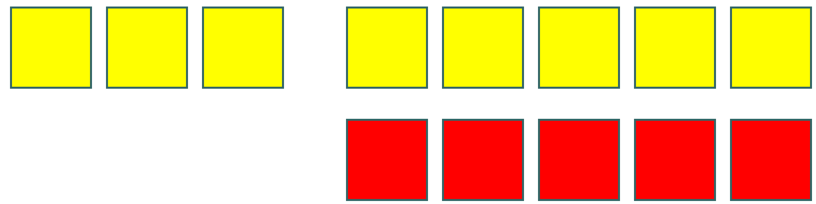
Subtraction of Integers

(+5) – (+2) = 

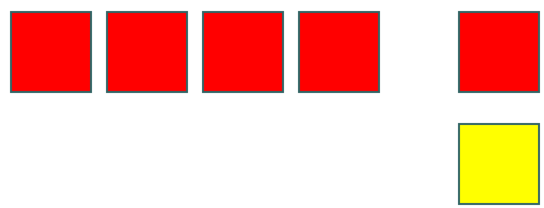
(-4) – (-3) = 

Subtracting Integers

(+3) – (-5)

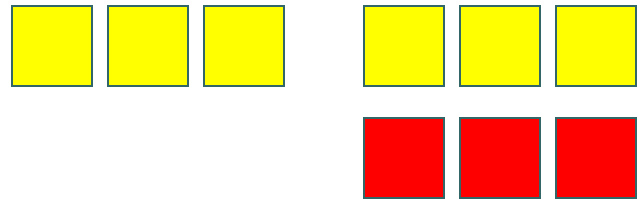


(-4) – (+1)



Subtracting Integers

(+3) – (-3)



- After students have seen many examples, have them formulate rules for integer subtraction.

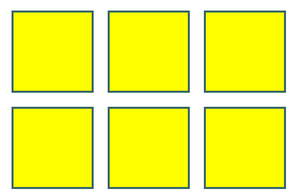
Multiplication of Integers

- Integer multiplication builds on whole number multiplication.
- Use concept that the multiplier serves as the "counter" of sets needed.
- For the given examples, use the algebra tiles to model the multiplication. Identify the multiplier or counter.
- Draw pictorial diagrams which model the multiplication process.

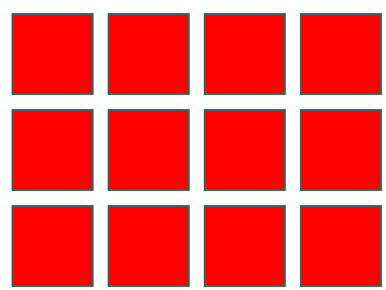
Multiplication of Integers

- The counter indicates how many rows to make. It has this meaning if it is positive.

(+2)(+3) =



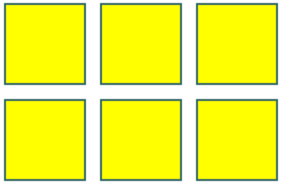
(+3)(-4) =



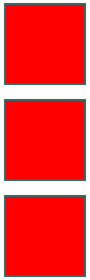
Multiplication of Integers

- If the counter is negative it will mean "take the opposite of." (flip-over)

(-2)(+3)



(-3)(-1)



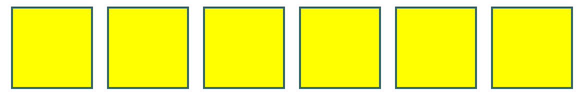
Multiplication of Integers

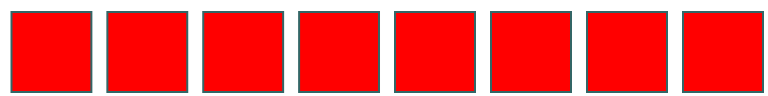
- After students have seen many examples, have them formulate rules for integer multiplication.
- Have students practice applying rules abstractly with larger integers.

Division of Integers

- Like multiplication, division relies on the concept of a counter.
- Divisor serves as counter since it indicates the number of rows to create.
- For the given examples, use algebra tiles to model the division. Identify the divisor or counter. Draw pictorial diagrams which model the process.

Division of Integers

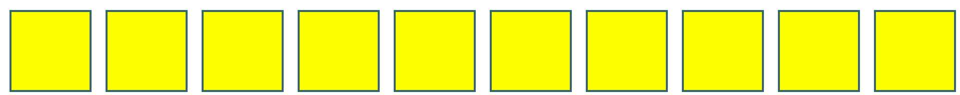
$(+6)/(+2) =$ 

$(-8)/(+2) =$ 

Division of Integers

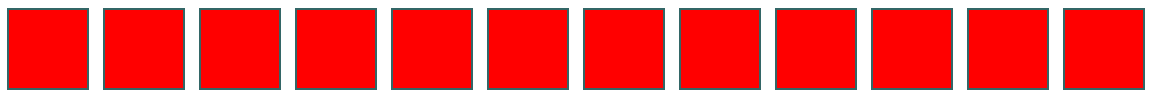
- A negative divisor will mean "take the opposite of." (flip-over)

(+10)/(-2) =



Division of Integers

(-12)/(-3) =



- After students have seen many examples, have them formulate rules.

Solving Equations

- Algebra tiles can be used to explain and justify the equation solving process. The development of the equation solving model is based on two ideas.
- Variables can be isolated by using zero pairs.
- Equations are unchanged if equivalent amounts are added to each side of the equation.

Solving Equations

- Use the green rectangle as X and the red rectangle (flip-side) as –X (the opposite of X).

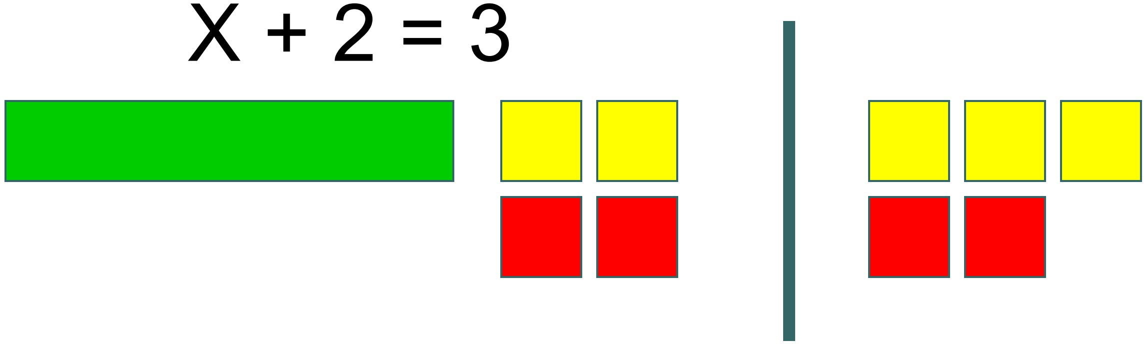
  $X + 2 = 3$

  $2X – 4 = 8$

  $2X + 3 = X – 5$

Solving Equations

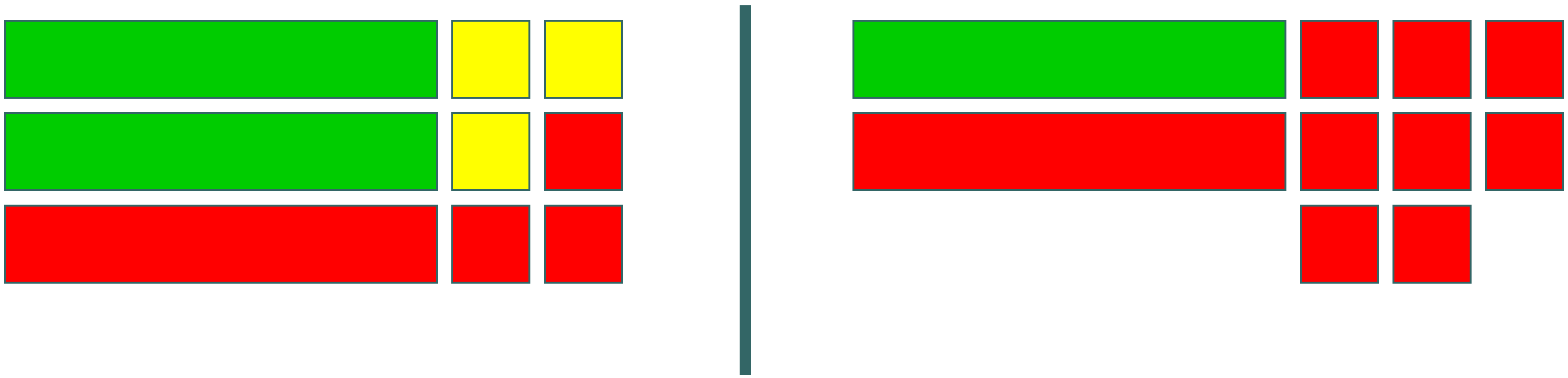
$X + 2 = 3$



$2X - 4 = 8$

Solving Equations

$$2X + 3 = X - 5$$



Distributive Property

- Use the same concept that was applied with multiplication of integers, think of the first factor as the counter.
- The same rules apply.

$3(X+2)$

- Three is the counter, so we need three rows of $(X+2)$

Distributive Property

3(X + 2)

3(X – 4)

-2(X + 2)

-3(X – 2)

Multiplication

- Multiplication using “base ten blocks.”
  (12)(13)
- Think of it as (10+2)(10+3)
- Multiplication using the array method allows students to see all four sub-products.

Modeling Polynomials

- Algebra tiles can be used to model expressions.
- Aid in the simplification of expressions.
- Add, subtract, multiply, divide, or factor polynomials.

Modeling Polynomials

- Let the blue square represent x^2, the green rectangle xy, and the yellow square y^2. The red square (flip-side of blue) represents $-x^2$, the red rectangle (flip-side of green) $-xy$, and the small red square (flip-side of yellow) $-y^2$.
- As with integers, the red shapes and their corresponding flip-sides form a zero pair.

Modeling Polynomials

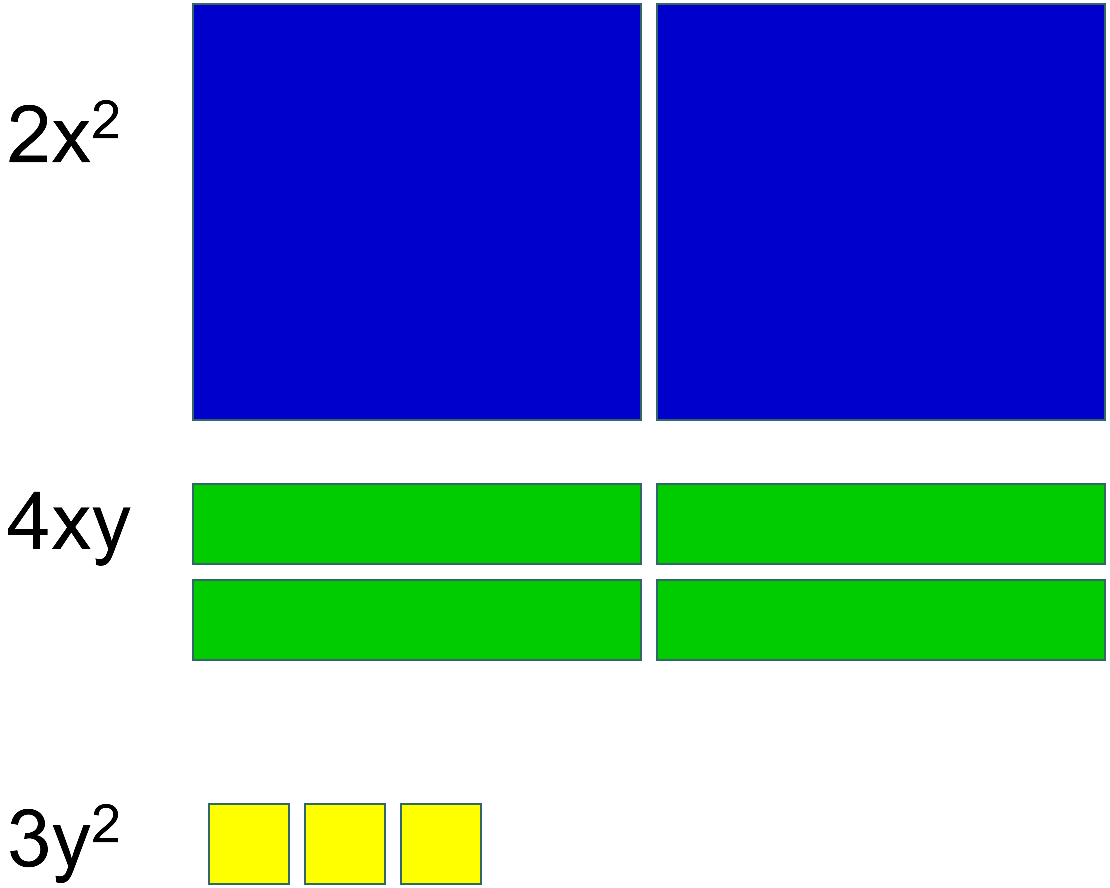
- Represent each of the following with algebra tiles, draw a pictorial diagram of the process, then write the symbolic expression.

  $2x^2$

  $4xy$

  $3y^2$

Modeling Polynomials



Modeling Polynomials

$3x^2 + 5y^2$

$-2xy$

$-3x^2 - 4xy$

- Textbooks do not always use x and y. Use other variables in the same format. Model these expressions.

$-a^2 + 2ab$

$5p^2 - 3pq + q^2$

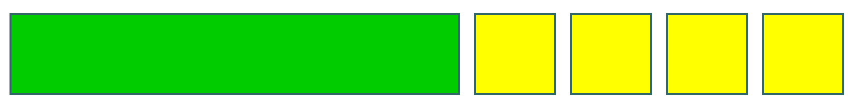
More Polynomials

- Would not present previous material and this information on the same day.
- Let the blue square represent x^2 and the large red square (flip-side) be $-x^2$.
- Let the green rectangle represent x and the red rectangle (flip-side) represent $-x$.
- Let yellow square represent 1 and the small red square (flip-side) represent -1.

More Polynomials

- Represent each of the given expressions with algebra tiles.
- Draw a pictorial diagram of the process.
- Write the symbolic expression.

x + 4



More Polynomials

$2x + 3$

$4x – 2$

More Polynomials

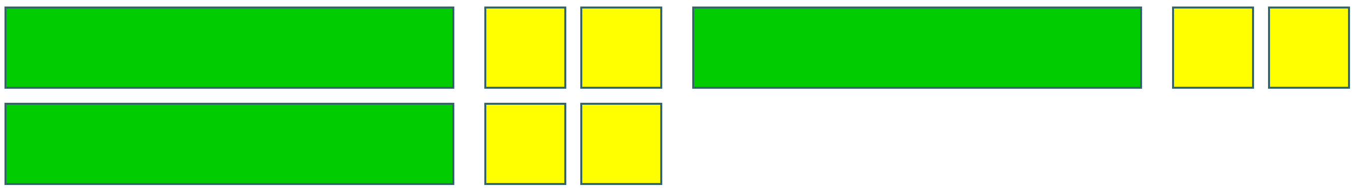
- Use algebra tiles to simplify each of the given expressions. Combine like terms. Look for zero pairs. Draw a diagram to represent the process.
- Write the symbolic expression that represents each step.

$2x + 4 + x + 2$

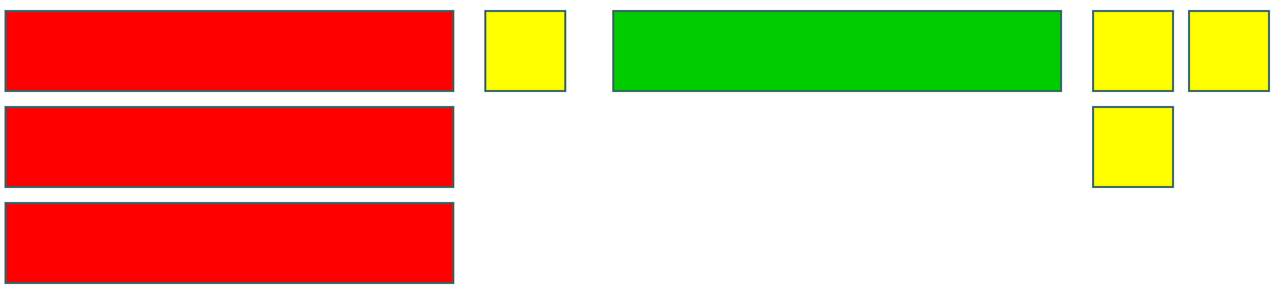
$-3x + 1 + x + 3$

More Polynomials

$2x + 4 + x + 2$



$-3x + 1 + x + 3$



More Polynomials

$3x + 1 - 2x + 4$

- This process can be used with problems containing x^2.

$(2x^2 + 5x - 3) + (-x^2 + 2x + 5)$

$(2x^2 - 2x + 3) - (3x^2 + 3x - 2)$

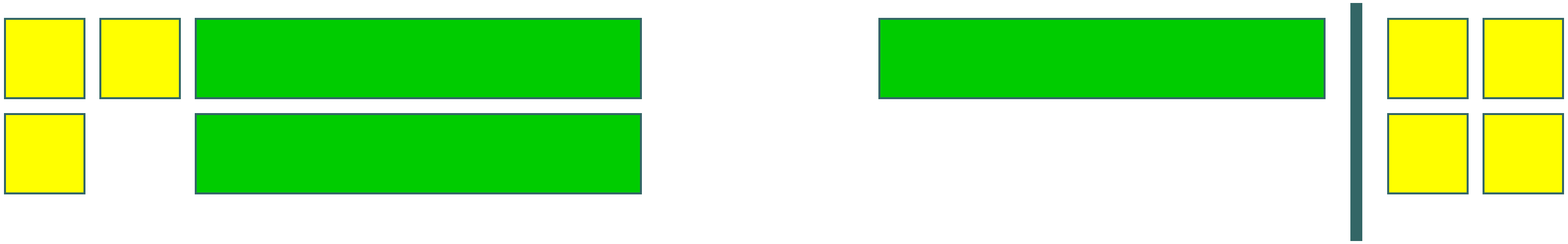
Substitution

- Algebra tiles can be used to model substitution. Represent original expression with tiles. Then replace each rectangle with the appropriate tile value. Combine like terms.

  $3 + 2x$ let $x = 4$

Substitution

3 + 2x   let x = 4

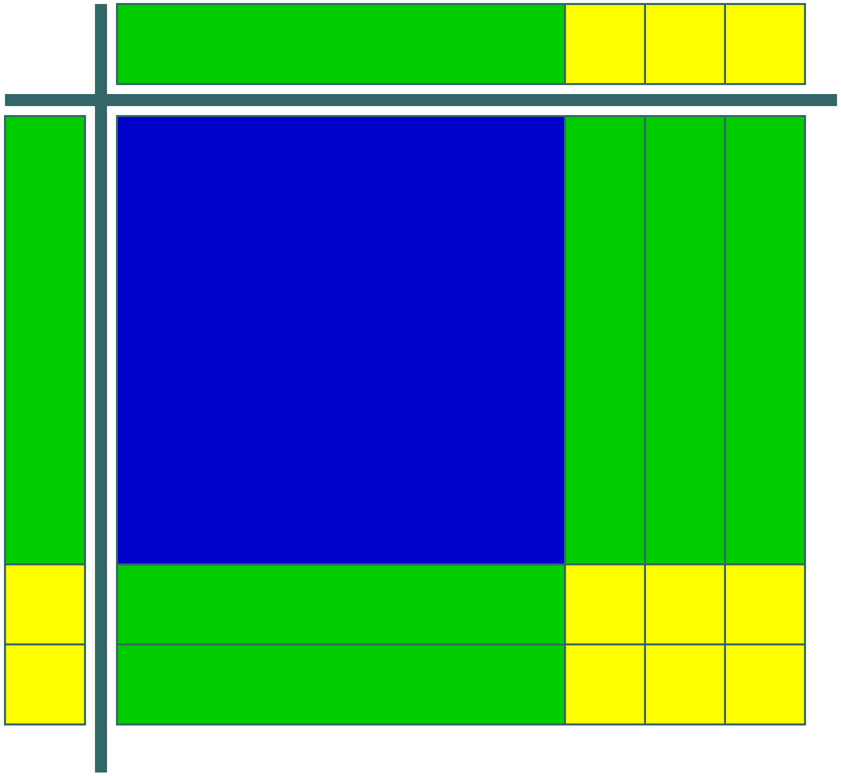


3 + 2x   let x = -4

3 – 2x   let x = 4

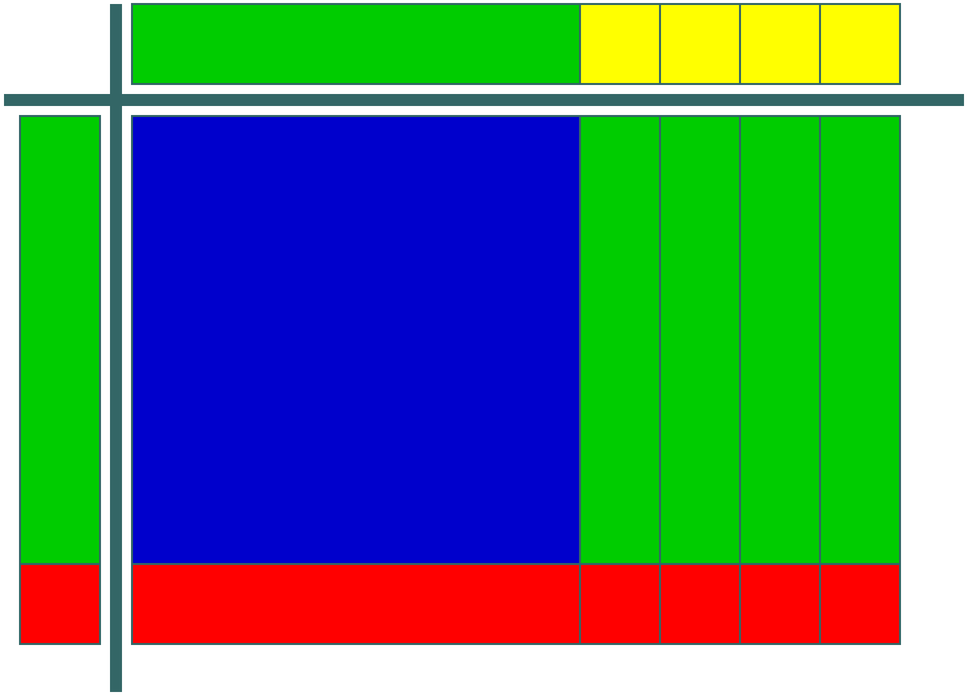
Multiplying Polynomials

(x + 2)(x + 3)



Multiplying Polynomials

(x – 1)(x +4)



Multiplying Polynomials

$(x + 2)(x - 3)$

$(x - 2)(x - 3)$

Factoring Polynomials

- Algebra tiles can be used to factor polynomials. Use tiles and the frame to represent the problem.
- Use the tiles to fill in the array so as to form a rectangle inside the frame.
- Be prepared to use zero pairs to fill in the array.
- Draw a picture.

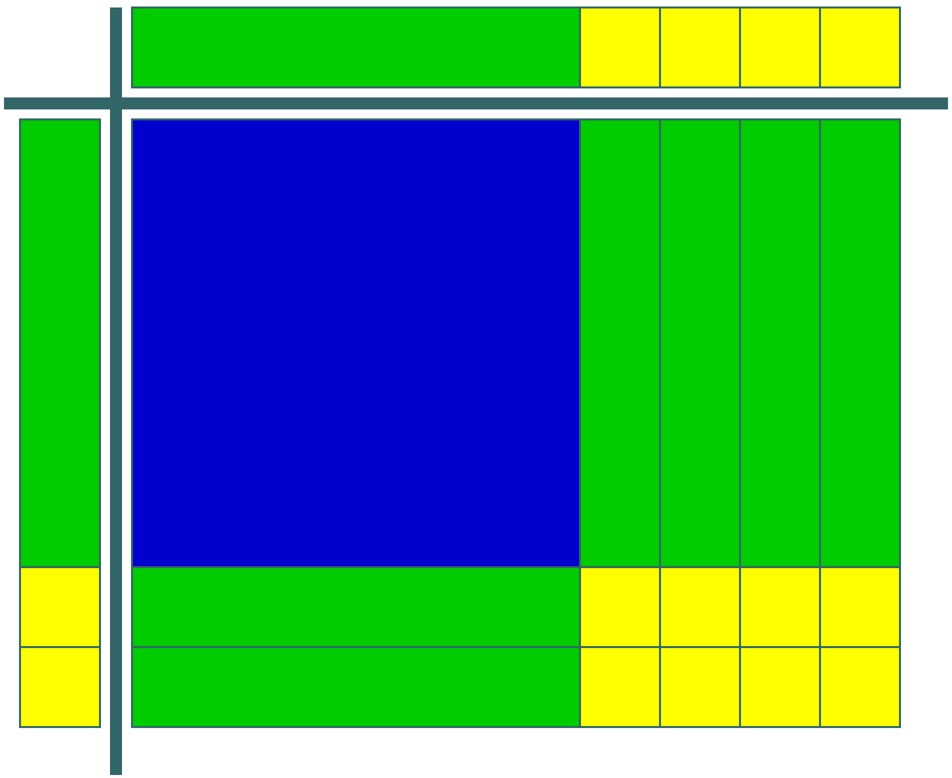
Factoring Polynomials

3x + 3

2x – 6

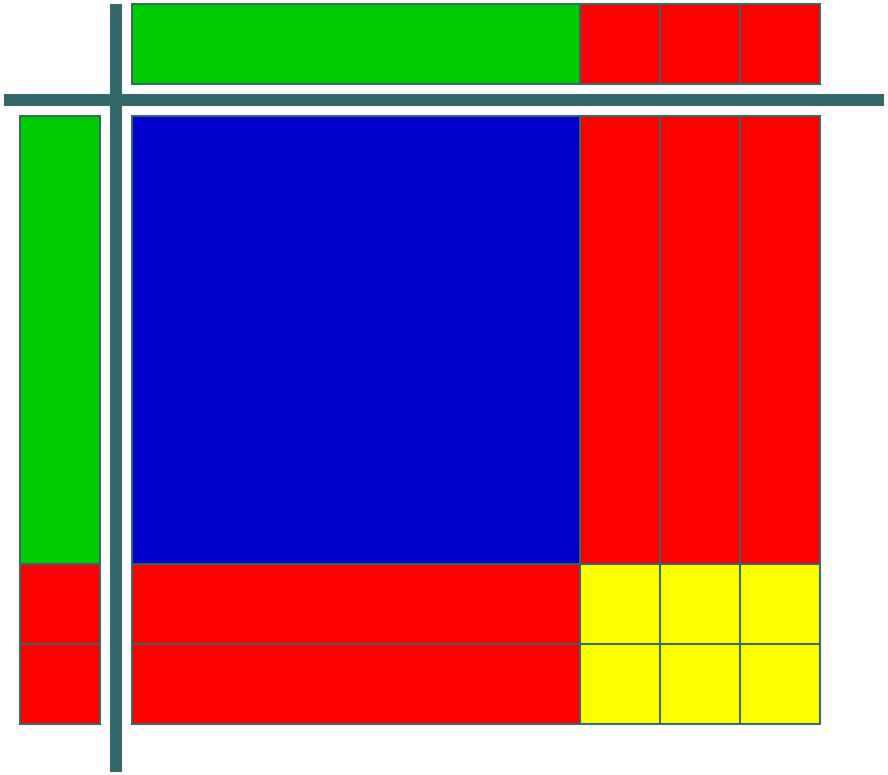
Factoring Polynomials

$x^2 + 6x + 8$



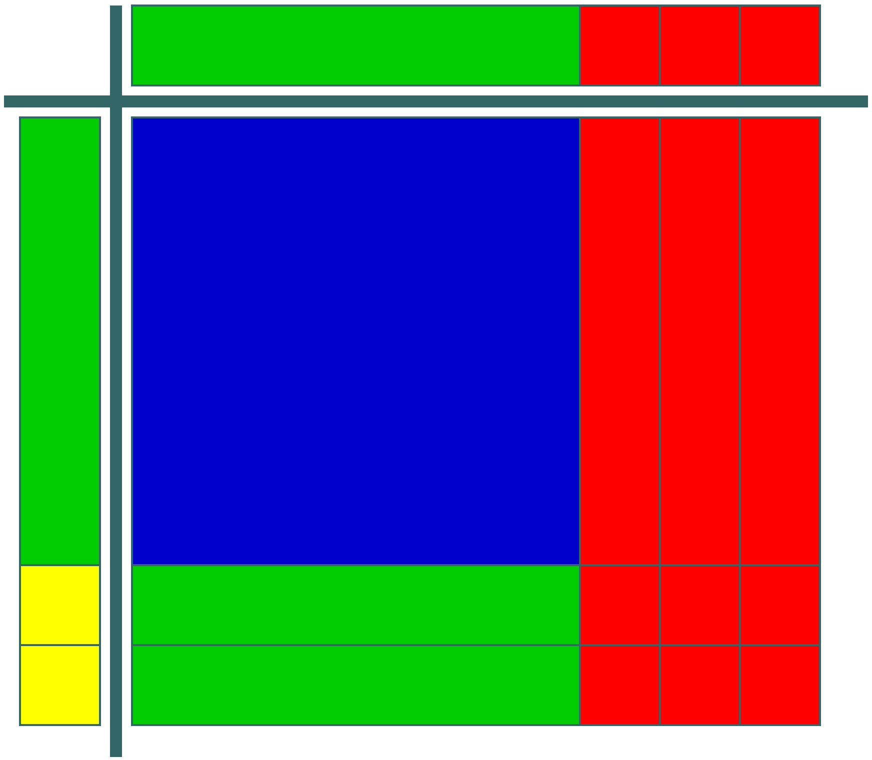
Factoring Polynomials

$x^2 - 5x + 6$



Factoring Polynomials

$x^2 - x - 6$



Factoring Polynomials

$x^2 + x - 6$

$x^2 - 1$

$x^2 - 4$

$2x^2 - 3x - 2$

$2x^2 + 3x - 3$

$-2x^2 + x + 6$

Dividing Polynomials

- Algebra tiles can be used to divide polynomials.
- Use tiles and frame to represent problem. Dividend should form array inside frame. Divisor will form one of the dimensions (one side) of the frame.
- Be prepared to use zero pairs in the dividend.

Dividing Polynomials

$$\frac{x^2 + 7x + 6}{x + 1}$$

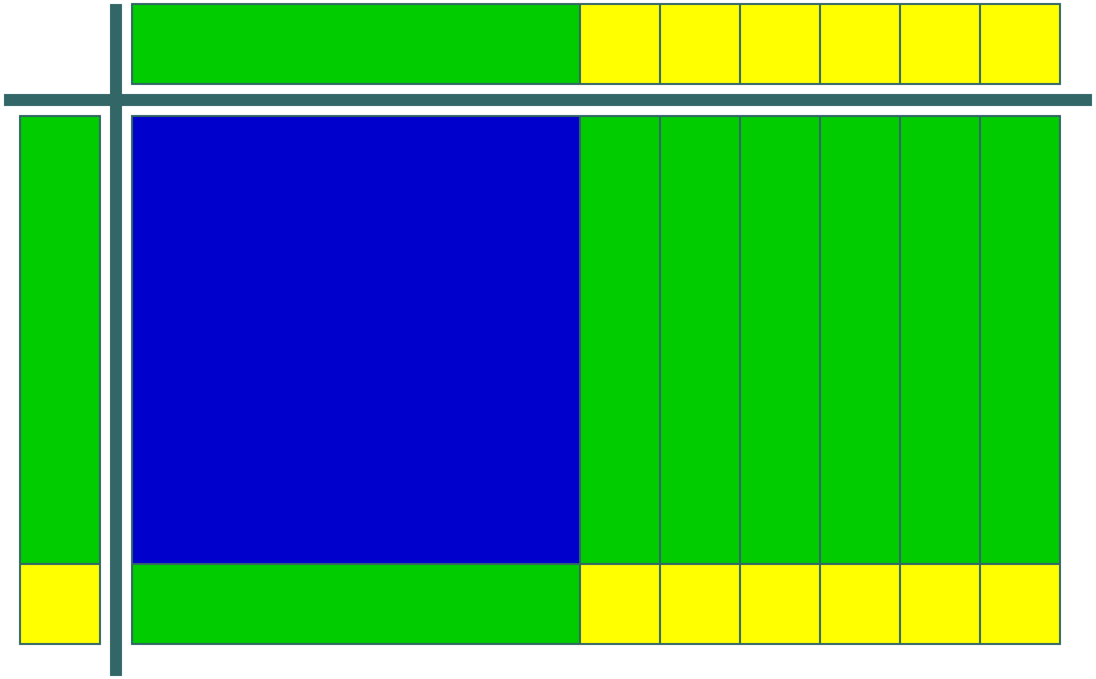
$$\frac{2x^2 + 5x - 3}{x + 3}$$

$$\frac{x^2 - x - 2}{x - 2}$$

$$\frac{x^2 + x - 6}{x + 3}$$

Dividing Polynomials

$$\frac{x^2 + 7x + 6}{x + 1}$$



Conclusion

"Polynomials are unlike the other "numbers" students learn how to add, subtract, multiply, and divide. They are not "counting" numbers. Giving polynomials a concrete reference (tiles) makes them real."

David A. Reid, Acadia University

Conclusion

- Algebra tiles can be made using the Ellison (die-cut) machine.
- On-line reproducible can be found by doing a search for algebra tiles.
- The TEKS that emphasize using algebra tiles are:

  Grade 7: 7.1(C), 7.2(C)

  Algebra I: c.3(B), c.4(B), d.2(A)

  Algebra II: c.2(E)

Conclusion

The Dana Center has several references to using algebra tiles in their Clarifying Activities. That site can be reached using:

http://www.tenet.edu/teks/math/clarifying/

Another way to get to the Clarifying Activities is by using the Dana Center's Math toolkit. That site is:

http://www.mathtekstoolkit.org